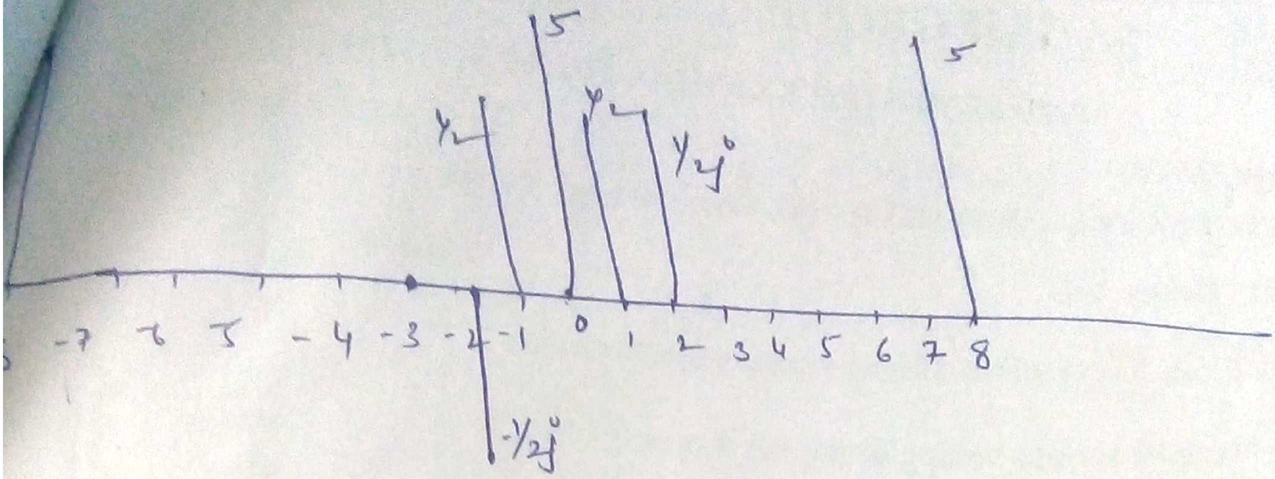
Freq. spectrum

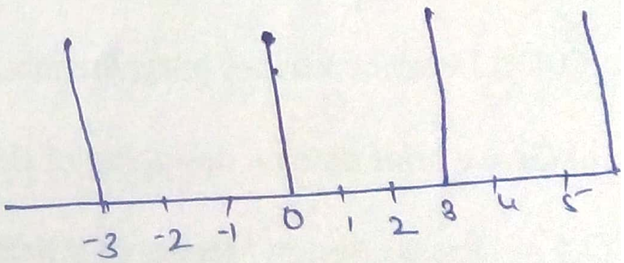


Q. Sketch the seq. $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-3k)$ and its DTFS.

Soln

$$x(n) = \sum_{k=-\infty}^{\infty} (\delta_{n-3k})$$

$$= \dots + \delta(n+6) + \delta(n+3) + \delta(n) + \delta(n-3) + \dots$$



$$C_k = \frac{1}{N} \sum_{0}^{N-1} x(n) \cdot e^{-j\pi 2kn/N}$$

$$= \frac{1}{3} \sum_{0}^{2} (x_n)\, e^{-j2\pi kn/3}$$

$$x(n) = \sum_{k=0}^{N-1} C_k\, e^{j2\pi kn/N}$$

$n=0$, $x_n = 1$, for $n=0$

$=0$, $n=1, 2$

$$C_k = \frac{1}{3} x(0) = 1/3 \text{ for}$$

all k.

Q. Consider the signal

$$x[n] = 1 + \sin\left(\frac{2\pi}{N}\right)n + 3\cos\left(\frac{2\pi}{N}\right)n + \cos\left(\frac{4\pi}{N}n + \pi/2\right)$$

$$x[n] = 1 + \frac{1}{2j}\left[e^{j\left(\frac{2\pi}{N}\right)n} - e^{-j(2\pi/N)n}\right] + \frac{3}{2}\left[e^{j(2\pi/N)n} + e^{-j(2\pi/N)n}\right]$$
$$+ \frac{1}{2}\left[e^{j(4\pi n/N + \pi/2)} + e^{-j(4\pi n/N + \pi/2)}\right]$$

$$= 1 + \frac{1}{2j}\left[e^{j\left(\frac{2\pi}{N}\right)n} - e^{-j(2\pi/N)n}\right] + \frac{3}{2}\left[e^{(j2\pi/N)n} + e^{-j(2\pi/N)n}\right]$$
$$+ \frac{1}{2}\left[\left(e^{j\pi/2}\right)e^{j2\left(\frac{2\pi}{N}\right)n} + \frac{1}{2}\left(e^{-j\pi/2}\right)e^{-j2(2\pi/N)n}\right]$$

$$= 1 + \left(\frac{3}{2} + \frac{1}{2j}\right)e^{\frac{j2\pi n}{N}} + \left(\frac{3}{2} - \frac{1}{2j}\right)e^{-j(2\pi/N)n} + \left(\frac{1}{2}e^{j\pi/2}\right)e^{j2\frac{2\pi}{N}n}$$
$$+ \frac{1}{2}\left(e^{-j\pi/2}\right)e^{-j2(2\pi/N)n}$$

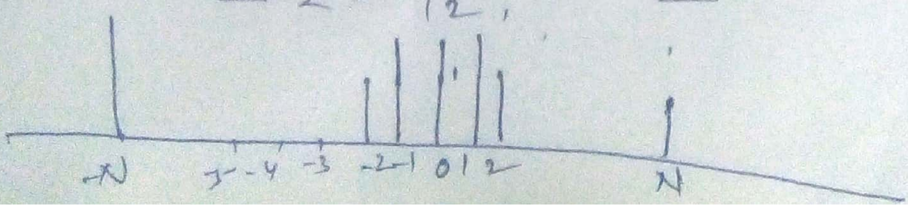
$\therefore\ a_0 = 1$, $\angle 0 = 0$

$a_1 = \frac{3}{2} + \frac{1}{2j} = \sqrt{10}/2$, $\quad a_{-1} = \left(\frac{3}{2} - \frac{1}{2j}\right) = \frac{\sqrt{10}}{2}$

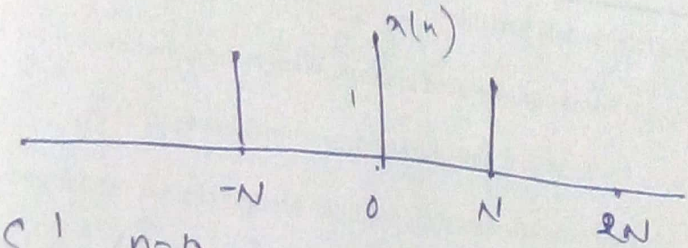
$a_2 = \frac{1}{2}j = \frac{1}{2}$ $\quad a_{-2} = -\frac{1}{2}j$ $\quad \frac{1}{2}$

$\angle a_0 = 0$, $\quad \angle a_1 = -\tan^{-1}(1/3)$, $\quad \angle a_{-1} = \tan^{-1}(1/3)$

$\angle a_2 = \pi/2$, $\quad \angle a_{-2} = -\pi/2$



find DTFS $x(n) = \sum_{m=-\infty}^{\infty} \delta(n-mN)$. $\delta(l)$



$\therefore x(n) = \begin{cases} 1 & n=0 \\ 0 & 1 \le n \le N-1 \end{cases}$

$= \delta(n)$.

fourier series coeff.

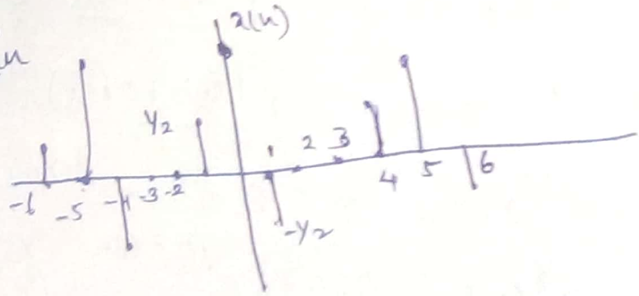
$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jk\omega_0 n}$

DFS: $x(n) = \sum_{k=0}^{k} C(k) e^{jk\omega_0 n}$.

$C(k) = \frac{1}{N} \sum_{n=0}^{N-1} \delta(n) e^{-j2k\pi n/N} = \frac{1}{N}\left[\sum_{n=0}^{N-1} \delta(n) e^{-j2k\left(\frac{1}{N}\right)\cdot n}\right]_{n=0}$

$= \frac{1}{N}$.

Q. find freq. domain



$N = 5$, $\omega_0 = \frac{2\pi}{N}$

odd symm.

$n = -2, \rightarrow 2$

$C_k = \frac{1}{N} \sum_{n=N} x(n) e^{-jk\omega_0 n} = \frac{1}{5} \sum_{n=-2}^{2} x(n) e^{-jk2\pi n/5}$

$= \frac{1}{5}\left[x(-2) e^{jk4\pi/5} + x(-1) e^{jk2\pi/5} + x(0) + x(1) e^{-jk2\pi/5} + x(2) e^{-jk\ldots}\right.$

using the value of $x(n)$

$C_k = \frac{1}{5}\left[0 + \frac{1}{2} e^{jk2\pi/5} + 1 - \frac{1}{2} e^{-jk2\pi/5}\right]$

$= \frac{1}{5}\left[1 + j\sin\left(k\frac{2\pi}{5}\right)\right]$